What are the Values of Multi-Dimensional Tic-Tac-Toe?

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In tic-tac-toe, you can rate a square's "real estate value" by how many possible three-in-a-rows pass through it. Thus, the center scores 4, the corners score 3 and the edges score 2. That means the center is the "most valuable" real estate, while the edges are the "least valuable."

Now, let's try three-dimensional tic-tac-toe, played on a 3-by-3-by-3 board. Which positions are the most and least valuable, and how much are they worth?

Extra credit: What about four-dimensional tic-tac-toe?

I took a brute force approach to this, representing every cell with *n*-dimensional coordinates, each valued 0, 1, or 2. A three dimensional corner might be (0, 2, 0), while the center cell would be (1, 1, 1). Then I looked at every possible pair of cells, and calculated the difference between their corresponding coordinates. If all of the differences were either 0's or 2's, then that pair of cells represented the end points of a winning row. I then tallied up all the winning rows through each cell. Here are the results.

3-Dimensional Tic-Tac-Toe

7	4	7
4	5	4
7	4	7

4	5	4
5	13	5
4	5	4

7	4	7
4	5	4
7	4	7

4-Dimensional Tic-Tac-Toe

15	8	15
8	7	8
15	8	15

8	7	8
7	14	7
8	7	8

15	8	15
8	7	8
15	8	15

8	7	8
7	14	7
8	7	8

7	14	7
14	40	14
7	14	7

8	7	8
7	14	7
8	7	8

15	8	15
8	7	8
15	8	15

8	7	8
7	14	7
8	7	8

15	8	15
8	7	8
15	8	15

5-Dimensional Tic-Tac-Toe

31	16	31
16	11	16
31	16	31

16	11	16
11	16	11
16	11	16

31	16	31
16	11	16
31	16	31

16	11	16
11	16	11
16	11	16

11	16	11
16	41	16
11	16	11

16	11	16
11	16	11
16	11	16

31	16	31
16	11	16
31	16	31

16	11	16
11	16	11
16	11	16

11	16	11
16	41	16
11	16	11

16	41	16
41	121	41
16	41	16

11	16	11
16	41	16
11	16	11

16	11	16
11	16	11
16	11	16

31	16	31
16	11	16
31	16	31

16	11	16
11	16	11
16	11	16

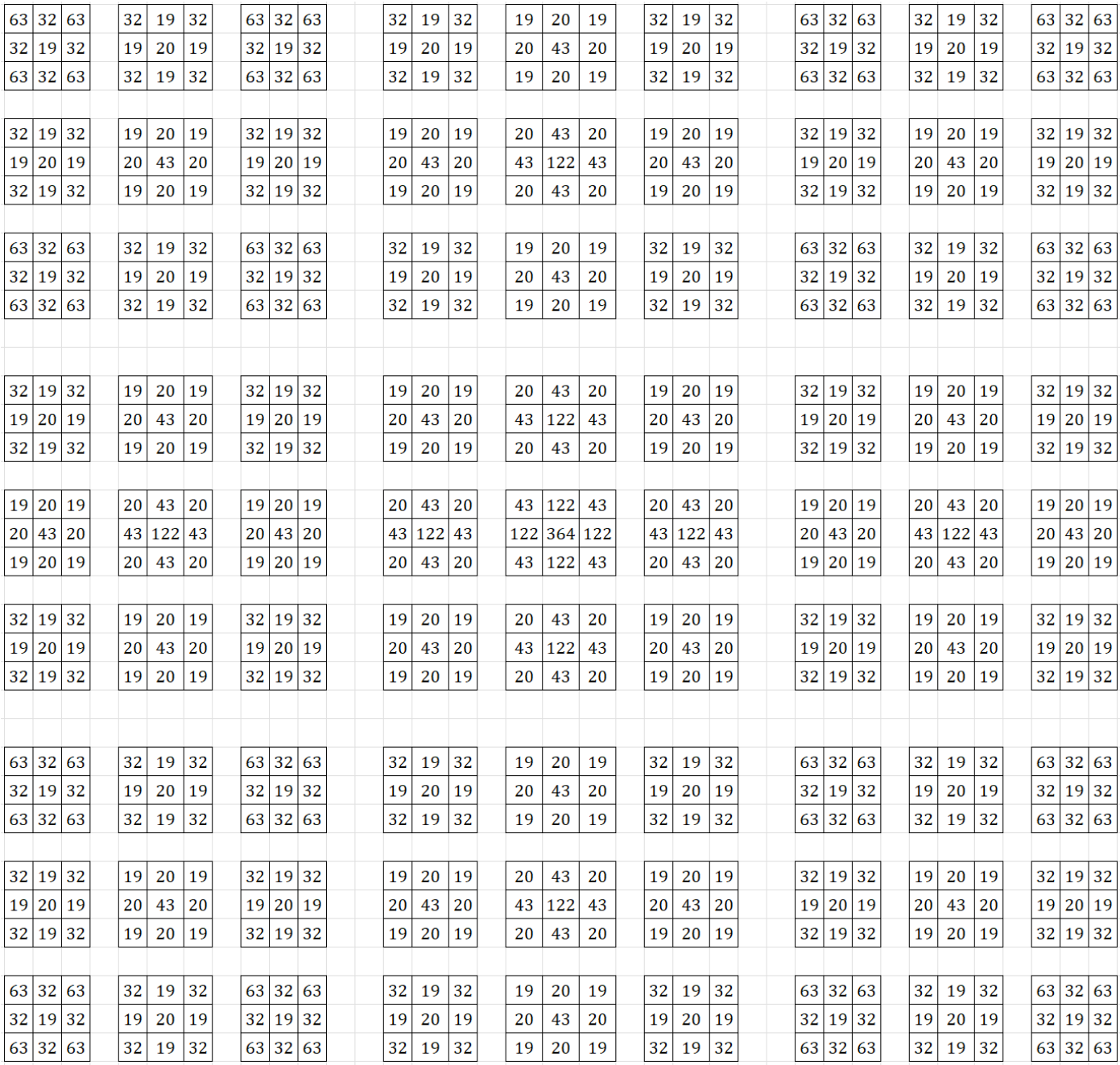
16	11	16
11	16	11
16	11	16

31	16	31
16	11	16
31	16	31

16	11	16
11	16	11
16	11	16

31	16	31
16	11	16
31	16	31

6-Dimensional Tic-Tac-Toe



Okay, that's enough.

A couple of generalizations: For any dimension *n*, the central cell will have a value of $(3^n - 1)/2$, and a corner cell (one with coordinate values of only 0 and 2) will have a value of $2^n - 1$.

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