What are the Values of Multi-Dimensional Tic-Tac-Toe?

Riddler Express of April 8, 2022

In tic-tac-toe, you can rate a square's "real estate value" by how many possible three-in-a-rows pass through it. Thus, the center scores 4, the corners score 3 and the edges score 2. That means the center is the "most valuable" real estate, while the edges are the "least valuable."

Now, let's try three-dimensional tic-tac-toe, played on a 3-by-3-by-3 board. Which positions are the most and least valuable, and how much are they worth?

Extra credit: What about four-dimensional tic-tac-toe?

I took a brute force approach to this, representing every cell with *n*-dimensional coordinates, each valued 0, 1, or 2. A three dimensional corner might be (0, 2, 0), while the center cell would be (1, 1, 1). Then I looked at every possible pair of cells, and calculated the difference between their corresponding coordinates. If all of the differences were either 0's or 2's, then that pair of cells represented the end points of a winning row. I then tallied up all the winning rows through each cell. Here are the results.

7	4	7	
4	5	4	
7	4	7	

3-Dim	ensional	Tic-Tac-Toe	e

5

13

5

4

5

4

4

5

4

7	4	7
4	5	4
7	4	7

4-Dimensional Tic-Tac-Toe

15	8	15
8	7	8
15	8	15

8	7	8
7	14	7
8	7	8

15	8	15
8	7	8
15	8	15

8	7	8
7	14	7
8	7	8

14	7
40	14
14	7
	40 14

		-
8	7	8
7	14	7
8	7	8

15	8	15
8	7	8
15	8	15

8	7	8
7	14	7
8	7	8

15	8	15
8	7	8
15	8	15

5-Dimensional	Tic-Tac-Toe

31	16	31	16	11	16	31	16	31
16	11	16	11	16	11	16	11	16
31	16	31	16	11	16	31	16	31
16	11	16	11	16	11	16	11	16
11	16	11	16	41	16	11	16	11
16	11	16	11	16	11	16	11	16
31	16	31	16	11	16	31	16	31
16	11	16	11	16	11	16	11	16
31	16	31	16	11	16	31	16	31
16	11	16	11	16	11	16	11	16
11	16	11	16	41	16	11	16	11
16	11	16	11	16	11	16	11	16
11	16	11	16	41	16	11	16	11
16	41	16	41	121	41	16	41	16
11	16	11	16	41	16	11	16	11
16	11	16	11	16	11	16	11	16
11	16	11	16	41	16	11	16	11
16	11	16	11	16	11	16	11	16
31	16	31	16	11	16	31	16	31
16	11	16	11	16	11	16	11	16
31	16	31	16	11	16	31	16	31
16	11	16	11	16	11	16	11	16
11	16	11	16	41	16	11	16	11
16	11	16	11	16	11	16	11	16
31	16	31	16	11	16	31	16	31
		10	11	16	11	16	11	16
16	11	16	11	10	11	10		10

6-Dimensional Tic-Tac-Toe

63	3 32	63	32	19	32	63	32	63	3	2 19	32	19	20	19	32	19	32	63	32	63		32	19	32	63	32	63
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63	3 32	63	32	19	32	63	32	<mark>63</mark>	33	2 19	32	19	20	19	32	19	32	63	32	63		32	19	32	63	32	63
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32	2 19	32	19	20	19	32	19	32	1	20	19	20	43	20	19	20	19	32	2 19	32		19	20	19	32	19	32
63	3 32	63	32	19	32	63	32	63	3	2 19	32	19	20	19	32	19	32	63	3 32	63		32	19	32	63	32	63

Okay, that's enough.

A couple of generalizations: For any dimension *n*, the central cell will have a value of $(3^n - 1)/2$, and a corner cell (one with coordinate values of only 0 and 2) will have a value of $2^n - 1$.

Dean Ballard April 10, 2022